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3. $g(x) = 2x + 5\cos x,\ [0, 2\pi]$

$g'(x) = 2 - 5\sin x$

$= 0$ when $\sin x = \frac{2}{5}$.

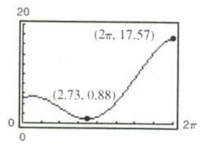
Critical numbers: $x \approx 0.41, x \approx 2.73$

Left endpoint: $(0, 5)$

Critical number: $(0.41, 5.41)$

Critical number: $(2.73, 0.88)$ Minimum

Right endpoint: $(2\pi, 17.57)$ Maximum



4. $f(x) = \dfrac{x}{\sqrt{x^2+1}},\ [0, 2]$

$$f'(x) = x\left[-\frac{1}{2}(x^2+1)^{-3/2}(2x)\right] + (x^2+1)^{-1/2}$$

$$= \frac{1}{(x^2+1)^{3/2}}$$

No critical numbers

Left endpoint: $(0, 0)$ Minimum

Right endpoint: $(2, 2/\sqrt{5})$ Maximum

5. Yes. $f(-3) = f(2) = 0$. f is continuous on $[-3, 2]$, differentiable on $(-3, 2)$.

$f'(x) = (x+3)(3x-1) = 0$ for $x = \frac{1}{3}$.

$c = \frac{1}{3}$ satisfies $f'(c) = 0$.

6. No. f is not differentiable at $x = 2$.

9. $f(x) = x^{2/3},\ 1 \le x \le 8$

$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$\frac{f(b)-f(a)}{b-a} = \frac{4-1}{8-1} = \frac{3}{7}$$

$$f'(c) = \frac{2}{3}c^{-1/3} = \frac{3}{7}$$

$$c = \left(\frac{14}{9}\right)^3 = \frac{2744}{729} \approx 3.764$$

10. $f(x) = \dfrac{1}{x},\ 1 \le x \le 4$

$$f'(x) = -\frac{1}{x^2}$$

$$\frac{f(b)-f(a)}{b-a} = \frac{(1/4)-1}{4-1} = \frac{-3/4}{3} = -\frac{1}{4}$$

$$f'(c) = \frac{-1}{c^2} = -\frac{1}{4}$$

$$c = 2$$

11. $f(x) = x - \cos x,\ -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$

$$f'(x) = 1 + \sin x$$

$$\frac{f(b)-f(a)}{b-a} = \frac{(\pi/2)-(-\pi/2)}{(\pi/2)-(-\pi/2)} = 1$$

$$f'(c) = 1 + \sin c = 1$$

$$c = 0$$

12. $f(x) = x\log_2 x = x \cdot \dfrac{\ln x}{\ln 2},\ [1, 2]$

$$f'(x) = \frac{1}{\ln 2}[\ln x + 1]$$

$$\frac{f(b)-f(a)}{b-a} = \frac{2-0}{2-1} = 2$$

$$f'(c) = 2 = \frac{1}{\ln 2}[\ln c + 1]$$

$$2\ln 2 - 1 = \ln c$$

$$c = e^{2\ln 2 - 1} = \frac{4}{e} \approx 1.4715$$

15. $f(x) = (x-1)^2(x-3)$

$f'(x) = (x-1)^2(1) + (x-3)(2)(x-1)$

$= (x-1)(3x-7)$

Critical numbers: $x = 1$ and $x = \frac{7}{3}$

Interval	$-\infty < x < 1$	$1 < x < \frac{7}{3}$	$\frac{7}{3} < x < \infty$
Sign of $f'(x)$	$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$
Conclusion	Increasing	Decreasing	Increasing

16. $g(x) = (x+1)^3$

$g'(x) = 3(x+1)^2$

Critical number: $x = -1$

Interval	$-\infty < x < -1$	$-1 < x < \infty$
Sign of $g'(x)$	$g'(x) > 0$	$g'(x) > 0$
Conclusion	Increasing	Increasing

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17. $h(x) = \sqrt{x}(x - 3) = x^{3/2} - 3x^{1/2}$

Domain: $(0, \infty)$

$h'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$

$= \frac{3}{2}x^{-1/2}(x - 1) = \frac{3(x - 1)}{2\sqrt{x}}$

Critical number: $x = 1$

Interval	$0 < x < 1$	$1 < x < \infty$
Sign of $h'(x)$	$h'(x) < 0$	$h'(x) > 0$
Conclusion	Decreasing	Increasing

18. $f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$

Critical numbers: $x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

Interval	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < 2\pi$
Sign of $f'(x)$	$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$
Conclusion	Increasing	Decreasing	Increasing

19. $f(t) = (2 - t)2^t$

$f'(t) = (2 - t)2^t \ln 2 - 2^t = 2^t[(2 - t)\ln 2 - 1]$

$f'(t) = 0{:}\ \ (2 - t)\ln 2 = 1$

$2 - t = \frac{1}{\ln 2}$

$t = 2 - \frac{1}{\ln 2} \approx 0.5573$, Critical number

Interval	$-\infty < t < 2 - \frac{1}{\ln 2}$	$2 - \frac{1}{\ln 2} < t < \infty$
Sign of $f'(t)$	$f'(t) > 0$	$f'(t) < 0$
Conclusion	Increasing	Decreasing

20. $g(x) = 2x \ln x$

$g'(x) = 2x\left(\frac{1}{x}\right) + 2\ln x = 2 + 2\ln x = 0$

$\ln x = -1$

Critical numbers: $x = \frac{1}{e}$

Test interval	$0 < x < \frac{1}{e}$	$\frac{1}{e} < x < \infty$
Sign of $g'(x)$	$g' < 0$	$g' > 0$
Conclusion	Decreasing	Increasing

21. $h(t) = \frac{1}{4}t^4 - 8t$

$h'(t) = t^3 - 8 = 0$ when $t = 2$.

Relative minimum: $(2, -12)$

Test interval	$-\infty < t < 2$	$2 < t < \infty$
Sign of $h'(t)$	$h'(t) < 0$	$h'(t) > 0$
Conclusion	Decreasing	Increasing

25. $f(x) = x + \cos x,\ 0 \le x \le 2\pi$

$f'(x) = 1 - \sin x$

$f''(x) = -\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

Points of inflection: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

Test interval	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 2\pi$
Sign of $f''(x)$	$f''(x) < 0$	$f''(x) > 0$	$f''(x) < 0$
Conclusion	Concave downward	Concave upward	Concave downward

26. $f(x) = (x + 2)^2(x - 4) = x^3 - 12x - 16$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x = 0$ when $x = 0$.

Point of inflection: $(0, -16)$

Test interval	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f''(x)$	$f''(x) < 0$	$f''(x) > 0$
Conclusion	Concave downward	Concave upward

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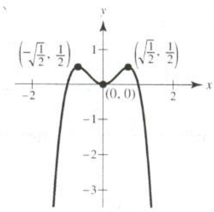
27. $g(x) = 2x^2(1 - x^2)$

$g'(x) = -4x(2x^2 - 1)$, Critical numbers: $x = 0, \pm\dfrac{1}{\sqrt{2}}$

$g''(x) = 4 - 24x^2$

$g''(0) = 4 > 0$, Relative minimum at $(0, 0)$

$g''\left(\pm\dfrac{1}{\sqrt{2}}\right) = -8 < 0$, Relative maximums at $\left(\pm\dfrac{1}{\sqrt{2}}, \dfrac{1}{2}\right)$



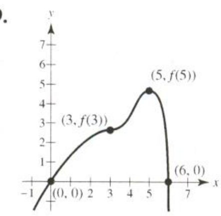
28. $h(t) = t - 4\sqrt{t + 1}$, Domain: $[-1, \infty)$

$h'(t) = 1 - \dfrac{2}{\sqrt{t + 1}} = 0 \Rightarrow t = 3$

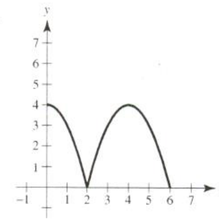
$h''(t) = \dfrac{1}{(t + 1)^{3/2}}$

$h''(3) = \dfrac{1}{8} > 0$, $(3, -5)$ is a relative minimum.

29.



30.



35. $\lim\limits_{x\to\infty} \dfrac{2x^2}{3x^2 + 5} = \dfrac{2}{3}$

36. $\lim\limits_{x\to\infty} \dfrac{2x}{3x^2 + 5} = 0$

37. $\lim\limits_{x\to-\infty} \dfrac{3x^2}{x + 5} = -\infty$

38. $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2 + x}}{-2x} = \dfrac{1}{2}$

39. $\lim\limits_{x\to\infty} \dfrac{5\cos x}{x} = 0$, since $|5\cos x| \le 5$.

40. $\lim\limits_{x\to\infty} \dfrac{3x}{\sqrt{x^2 + 4}} = \lim\limits_{x\to\infty} \dfrac{3}{\sqrt{1 + 4/x^2}} = 3$

41. $\lim\limits_{x\to-\infty} \dfrac{6x}{x + \cos x} = 6$

42. $\lim\limits_{x\to-\infty} \dfrac{x}{2\sin x}$ does not exist.

43. $h(x) = \dfrac{2x + 3}{x - 4}$

Discontinuity: $x = 4$

$$\lim_{x\to\infty} \frac{2x + 3}{x - 4} = \lim_{x\to\infty} \frac{2 + (3/x)}{1 - (4/x)} = 2$$

Vertical asymptote: $x = 4$

Horizontal asymptote: $y = 2$

44. $g(x) = \dfrac{5x^2}{x^2 + 2}$

$$\lim_{x\to\infty} \frac{5x^2}{x^2 + 2} = \lim_{x\to\infty} \frac{5}{1 + (2/x^2)} = 5$$

Horizontal asymptote: $y = 5$

45. $f(x) = \dfrac{3}{x} - 2$

Discontinuity: $x = 0$

$$\lim_{x\to\infty} \left(\frac{3}{x} - 2\right) = -2$$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = -2$

46. $f(x) = \dfrac{3x}{\sqrt{x^2 + 2}}$

$$\lim_{x\to\infty} \frac{3x}{\sqrt{x^2 + 2}} = \lim_{x\to\infty} \frac{3x/x}{\sqrt{x^2 + 2}/\sqrt{x^2}}$$

$$= \lim_{x\to\infty} \frac{3}{\sqrt{1 + (2/x^2)}} = 3$$

$$\lim_{x\to-\infty} \frac{3x}{\sqrt{x^2 + 2}} = \lim_{x\to-\infty} \frac{3x/x}{\sqrt{x^2 + 2}/\left(-\sqrt{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{3}{-\sqrt{1 + (2/x^2)}} = -3$$

Horizontal asymptotes: $y = \pm 3$

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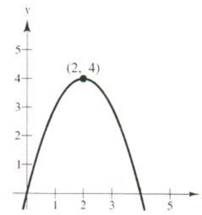
55. $f(x) = 4x - x^2 = x(4 - x)$



Domain: $(-\infty, \infty)$; Range: $(-\infty, 4)$

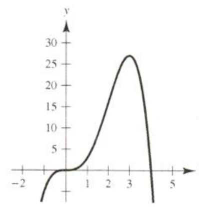
$f'(x) = 4 - 2x = 0$ when $x = 2$.

$f''(x) = -2$

Therefore, $(2, 4)$ is a relative maximum.

Intercepts: $(0, 0), (4, 0)$

56. $f(x) = 4x^3 - x^4 = x^3(4 - x)$



Domain: $(-\infty, \infty)$; Range: $(-\infty, 27)$

$f'(x) = 12x^2 - 4x^3 = 4x^2(3 - x) = 0$ when $x = 0, 3$.

$f''(x) = 24x - 12x^2 = 12x(2 - x) = 0$ when $x = 0, 2$.

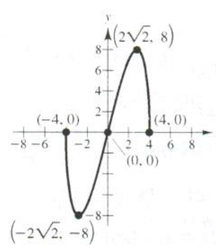
$f''(3) < 0$

Therefore, $(3, 27)$ is a relative maximum.

Points of inflection: $(0, 0), (2, 16)$

Intercepts: $(0, 0), (4, 0)$

57. $f(x) = x\sqrt{16 - x^2}$



Domain: $[-4, 4]$; Range: $[-8, 8]$

$f'(x) = \dfrac{16 - 2x^2}{\sqrt{16 - x^2}} = 0$ when $x = \pm 2\sqrt{2}$ and undefined when $x = \pm 4$.

$f''(x) = \dfrac{2x(x^2 - 24)}{(16 - x^2)^{3/2}}$

$f''(-2\sqrt{2}) > 0$

Therefore, $(-2\sqrt{2}, -8)$ is a relative minimum.

$f''(2\sqrt{2}) < 0$

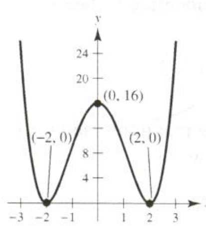
Therefore, $(2\sqrt{2}, 8)$ is a relative maximum.

Point of inflection: $(0, 0)$

Intercepts: $(-4, 0), (0, 0), (4, 0)$

Symmetry with respect to origin

58. $f(x) = (x^2 - 4)^2$



Domain: $(-\infty, \infty)$; Range: $[0, \infty)$

$f'(x) = 4x(x^2 - 4) = 0$ when $x = 0, \pm 2$.

$f''(x) = 4(3x^2 - 4) = 0$ when $x = \pm\dfrac{2\sqrt{3}}{3}$.

$f''(0) < 0$

Therefore, $(0, 16)$ is a relative maximum.

$f''(\pm 2) > 0$

Therefore, $(\pm 2, 0)$ are relative minima.

Points of inflection: $(\pm 2\sqrt{3}/3, 64/9)$

Intercepts: $(-2, 0), (0, 16), (2, 0)$

Symmetry with respect to y-axis

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59. $f(x) = (x-1)^3(x-3)^2$

Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

$f'(x) = (x-1)^2(x-3)(5x-11) = 0$ when $x = 1, \frac{11}{5}, 3.$

$f''(x) = 4(x-1)(5x^2-22x+23) = 0$ when $x = 1, \frac{11 \pm \sqrt{6}}{5}.$

$f''(3) > 0$

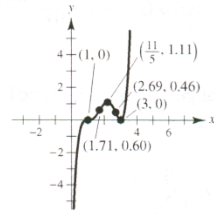
Therefore, $(3, 0)$ is a relative minimum.

$f''\left(\frac{11}{5}\right) < 0$

Therefore, $\left(\frac{11}{5}, \frac{3456}{3125}\right)$ is a relative maximum.

Points of inflection: $(1, 0), \left(\frac{11-\sqrt{6}}{5}, 0.60\right), \left(\frac{11+\sqrt{6}}{5}, 0.46\right)$

Intercepts: $(0, -9), (1, 0), (3, 0)$



60. $f(x) = (x-3)(x+2)^3$

Domain: $(-\infty, \infty)$; Range: $\left[-\frac{16,875}{256}, \infty\right)$

$f'(x) = (x-3)(3)(x+2)^2 + (x+2)^3$

$= (4x-7)(x+2)^2 = 0$ when $x = -2, \frac{7}{4}.$

$f''(x) = (4x-7)(2)(x+2) + (x+2)^2(4)$

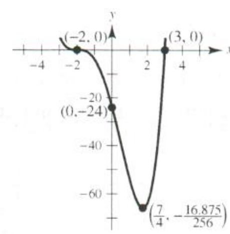
$= 6(2x-1)(x+2) = 0$ when $x = -2, \frac{1}{2}.$

$f''\left(\frac{7}{4}\right) > 0$

Therefore, $\left(\frac{7}{4}, -\frac{16,875}{256}\right)$ is a relative minimum.

Points of inflection: $(-2, 0), \left(\frac{1}{2}, -\frac{625}{16}\right)$

Intercepts: $(-2, 0), (0, -24), (3, 0)$



61. $f(x) = x^{1/3}(x+3)^{2/3}$

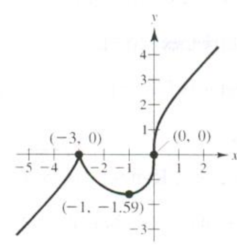
Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

$f'(x) = \frac{x+1}{(x+3)^{1/3}x^{2/3}} = 0$ when $x = -1$ and undefined when $x = -3, 0.$

$f''(x) = \frac{-2}{x^{5/3}(x+3)^{4/3}}$ is undefined when $x = 0, -3.$

By the First Derivative Test $(-3, 0)$ is a relative maximum and $\left(-1, -\sqrt[3]{4}\right)$ is a relative minimum. $(0, 0)$ is a point of inflection.

Intercepts: $(-3, 0), (0, 0)$



63. $f(x) = \frac{x+1}{x-1}$

Domain: $(-\infty, 1), (1, \infty)$; Range: $(-\infty, 1), (1, \infty)$

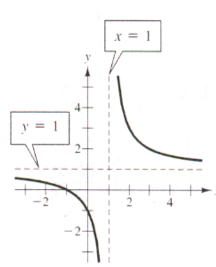
$f'(x) = \frac{-2}{(x-1)^2} < 0$ if $x \neq 1.$

$f''(x) = \frac{4}{(x-1)^3}$

Horizontal asymptote: $y = 1$

Vertical asymptote: $x = 1$

Intercepts: $(-1, 0), (0, -1)$



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64. $f(x) = \dfrac{2x}{1 + x^2}$

Domain: $(-\infty, \infty)$; Range: $[-1, 1]$

$f'(x) = \dfrac{2(1 - x)(1 + x)}{(1 + x^2)^2} = 0$ when $x = \pm 1$.

$f''(x) = \dfrac{-2x(3 - x^2)}{(1 + x^2)^3} = 0$ when $x = 0, \pm\sqrt{3}$.

$f''(1) < 0$

Therefore, $(1, 1)$ is a relative maximum.

$f''(-1) > 0$

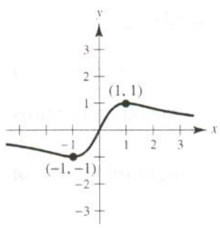
Therefore, $(-1, -1)$ is a relative minimum.

Points of inflection: $(-\sqrt{3}, -\sqrt{3}/2), (0, 0), (\sqrt{3}, \sqrt{3}/2)$

Intercept: $(0, 0)$

Symmetric with respect to the origin

Horizontal asymptote: $y = 0$



65. $f(x) = \dfrac{4}{1 + x^2}$

Domain: $(-\infty, \infty)$; Range: $(0, 4]$

$f'(x) = \dfrac{-8x}{(1 + x^2)^2} = 0$ when $x = 0$.

$f''(x) = \dfrac{-8(1 - 3x^2)}{(1 + x^2)^3} = 0$ when $x = \pm\dfrac{\sqrt{3}}{3}$.

$f''(0) < 0$

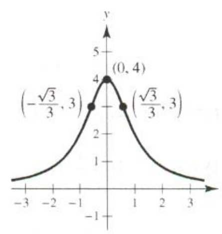
Therefore, $(0, 4)$ is a relative maximum.

Points of inflection: $(\pm\sqrt{3}/3, 3)$

Intercept: $(0, 4)$

Symmetric to the y-axis

Horizontal asymptote: $y = 0$



67. $f(x) = x^3 + x + \dfrac{4}{x}$

Domain: $(-\infty, 0), (0, \infty)$; Range: $(-\infty, -6], [6, \infty)$

$f'(x) = 3x^2 + 1 - \dfrac{4}{x^2} = \dfrac{3x^4 + x^2 - 4}{x^2} = 0$ when $x = \pm 1$.

$f''(x) = 6x + \dfrac{8}{x^3} = \dfrac{6x^4 + 8}{x^3} \neq 0$

$f''(-1) < 0$

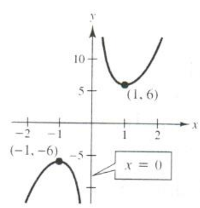
Therefore, $(-1, -6)$ is a relative maximum.

$f''(1) > 0$

Therefore, $(1, 6)$ is a relative minimum.

Vertical asymptote: $x = 0$

Symmetric with respect to origin



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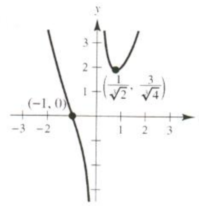
68. $f(x) = x^2 + \dfrac{1}{x} = \dfrac{x^3 + 1}{x}$



Domain: $(-\infty, 0), (0, \infty)$; Range: $(-\infty, \infty)$

$f'(x) = 2x - \dfrac{1}{x^2} = \dfrac{2x^3 - 1}{x^2} = 0$ when $x = \dfrac{1}{\sqrt[3]{2}}$.

$f''(x) = 2 + \dfrac{2}{x^3} = \dfrac{2(x^3 + 1)}{x^3} = 0$ when $x = -1$.

$f''\left(\dfrac{1}{\sqrt[3]{2}}\right) > 0$

Therefore, $\left(\dfrac{1}{\sqrt[3]{2}}, \dfrac{3}{\sqrt[3]{4}}\right)$ is a relative minimum.

Point of inflection: $(-1, 0)$

Intercept: $(-1, 0)$

Vertical asymptote: $x = 0$

71. $h(x) = (1 - x)e^x$

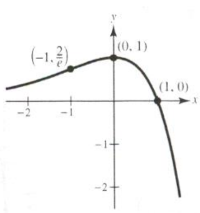
$h'(x) = -xe^x$

$h''(x) = -(x + 1)e^x$

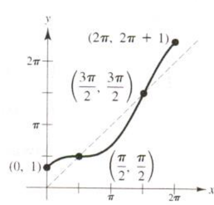
Horizontal asymptote: $y = 0$ (to the left)

Critical point: $(0, 1)$ (relative maximum)

Inflection point: $(-1, 2/e) \approx (-1, 0.736)$



77. $f(x) = x + \cos x$



Domain: $[0, 2\pi]$; Range: $[1, 1 + 2\pi]$

$f'(x) = 1 - \sin x \geq 0$, f is increasing.

$f''(x) = -\cos x = 0$ when $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$.

Points of inflection: $\left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right), \left(\dfrac{3\pi}{2}, \dfrac{3\pi}{2}\right)$

Intercept: $(0, 1)$